Two-Third power law for curved movements provides a strict constraint on optimal control models

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One of the interesting properties of curved hand movements is that the speed v is related to the curvature k through a power law: $v(t) \propto k(t)^{-1/3}$, called the ⅔ power law (because the angular speed is proportional to $k^{2/3}$).

Several models have been proposed to explain the origin of the ⅔ power law from optimality principle: the minimum jerk models (Richardson and Flash 2002; Todorov and Jordan 1998; Viviani and Flash 1995; Wann et al. 1988), and the minimum-variance model (Harris and Wolpert 1998), to name a few. It seemed the ⅔ power law was an easily explainable phenomenon.

In further analyzing these models, however, we found their predictions to hold only for movement paths that have rather quickly changing curvature (such as ellipses, cloverleaf shapes and figure eights). For paths with slowly changing curvature, such as spirals, the power law is no longer ⅔.

This failure can be analytically proved for the minimum jerk model. We show that the minimum jerk model in fact predicts a ⅓ power law ($v(t) \propto k(t)^{-2/3}$) for paths with slowly changing curvature, and that the power law approaches ⅔ for paths with fast changing curvature.

Our analysis shows that predicting the ⅔ power law for a wide range of paths is a very strict criterion that narrows down the appropriate cost function. We are currently investigating a family of cost functions that satisfy this criterion.