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A THEORETICAL ANALYSIS OF METHODS OF INTERPRETING RADIO-LINE DATA FROM H II REGIONS

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ABSTRACT

The best form of analysis of data on radio recombination lines from H II regions is shown to require inclusion of both non-LTE effects and highly concentrated clumping. The principal physical effect of clumping is to reduce considerably the line enhancements predicted by the non-LTE theory. Theoretically, all radio recombination lines emitted by the same mass of ionized gas should, within the error limits, be interpretable in terms of a single electron temperature.

I. INTRODUCTION

The theoretical interpretation of data on radio recombination lines emitted by H II regions is beset by two principal complications: deviations from local thermodynamic equilibrium (LTE) in the line-formation processes; and variations in electron temperature (T_e) and electron concentration (N_e) along a line of sight through an H II region. The importance of these effects is difficult to evaluate by a discussion of observational data, because one deals with uncertainties both in the theory to be applied and in the data. The purpose of the present paper is to use a completely theoretical analysis to show the extent to which non-LTE and structure effects should be important. The method used involves three steps: (1) ionization and temperature variations are calculated under the assumption of an arbitrary density distribution for the line of sight through the center of a nebulae; (2) intensities in the continuum and several recombination lines (H94α, H109α, H126α, H158α, H137β, and H225γ) are calculated from equations of radiative transfer by using the data on variations in T_e and N_e together with data on the non-LTE populations of the energy levels taken from Sejnowski and Hjellming (1969); and (3) calculated line and continuum intensities are then treated as if they were obtained as observational data and are subjected to both LTE and non-LTE analysis to determine temperature, which can then be compared with the actual average temperatures for the theoretical models. With this procedure there are no limitations due to errors inherent in observational data or lack of knowledge of the actual structure.

It is shown that, although the LTE analysis gives approximately correct results for model nebulae with small emission measures ($E \lesssim 10^5$ pc cm^{-6}), for larger emission measures the non-LTE effects are very important; and, largely because of the non-LTE line enhancement at the larger emission measures, the introduction of clumping into the models is essential in reducing the line enhancement to the strengths generally observed. It is concluded that all analysis of data on radio recombination lines should be made with a non-LTE theory in which there are only three parameters for each line of sight: the electron temperature, the emission measure, and the average electron con-

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centration, $\langle N_e \rangle_E$, obtained by averaging with a weighting according to the contribution to the emission measure. With this form of analysis, all radio recombination lines should, within the error limits inherent in the observational data and the theoretical assumptions, allow the determination of the same temperature, which will be close to $\langle T_e \rangle_E$, the average temperature calculated with a weighting according to the contribution to the emission measure.

In § II the formation of radio recombination lines is discussed. In § III the method of computation of ionization and temperature variations is presented, and in § IV the associated line and continuum intensities are calculated. The effects of different methods of determining temperatures from the line strengths are discussed in § V, and conclusions are presented in § VI.

II. FORMATION OF LINES AND CONTINUUM

If we consider a particular radio recombination line and denote quantities referring to the line by a subscript L and quantities referring to the adjacent continuum by a subscript C, then the brightness temperatures in the line and in the continuum, T_L and T_C, are determined from the standard radiative-transfer solution (Chandrasekhar 1960; Kraus 1966), giving

$$T_L + T_C = \int_0^s \frac{\lambda^2}{2k}(j_L + j_C)\rho \exp[-\tau_L(s') - \tau_C(s')]ds' \tag{1}$$

and

$$T_C = \int_0^s \frac{\lambda^2}{2k} j_C \rho \exp[-\tau_C(s')]ds' , \tag{2}$$

where

$$\tau_L(s') = \int_0^{s'} \kappa_L \rho ds'' \tag{3}$$

and

$$\tau_C(s') = \int_0^{s'} \kappa_C \rho ds'' . \tag{4}$$

In these equations λ is the wavelength, k is the Boltzmann constant, ρ is the mass density, j_L and j_C are the mass emission coefficients in the line and in the continuum, respectively, and κ_L and κ_C are the associated mass absorption coefficients. Integrals are evaluated along a line of sight through the nebula from the far side, where $s' = 0$, to the near side, where $s' = s$. The quantities τ_L and τ_C are the optical depths due to line and continuum processes, respectively. We neglect all background sources of radiation.

Clearly, the emission and absorption coefficients used in equations (1)–(4) will determine the solution for the brightness temperatures. For the continuum coefficients we will use the expression given by Oster (1961) for free-free absorption:

$$\kappa_C \rho = (0.9768 \times 10^{-20}) \left(\frac{T_e}{°\mathrm{K}}\right)^{-1.5} \left(\frac{\nu}{\mathrm{GHz}}\right)^{-2.0} \left[\ln 0.04955 \left(\frac{\nu}{\mathrm{GHz}}\right)^{-1} + 1.5 \ln \left(\frac{T_e}{°\mathrm{K}}\right)\right] N_i N_e \ \mathrm{cm}^{-1} ; \tag{5}$$

$j_C \rho$ can be derived from the equation for detailed balancing, $j_C = (2kT_e/\lambda^2)\kappa_C$. In equation (5), N_i is the ion concentration, which we will consider to be equal to N_e.

If one uses the standard theory of spectral-line formation (see Jeffries 1968), one can show that, for transitions between a lower level of quantum number n and a higher level of quantum number m, the coefficients for a line formed in LTE are:

$$\kappa_L^{\mathrm{LTE}}\rho = \frac{\lambda^2}{8\pi}\frac{g_m}{g_n}A_{mn}\left[1 - \exp\left(-\frac{h\nu}{kT_e}\right)\right]N_n^{\mathrm{LTE}}\phi_{nm}(\nu) \tag{6}$$

and

$$j_L^{\mathrm{LTE}}\rho = N_m^{\mathrm{LTE}}A_{mn}\frac{h\nu}{4\pi}\phi_{nm}(\nu)\ , \tag{7}$$

where g_m and g_n are statistical weights ($g_n = 2n^2$ for hydrogen), A_{mn} is the Einstein coefficient for spontaneous emission, $h\nu$ is the energy difference involved in the $m \rightarrow n$ transition, and N_n^{LTE} is the LTE population of the level n, which is determined by the Eggert-Saha equation:

$$N_n^{\mathrm{LTE}} = N(\mathrm{H\ II})N_e\left(\frac{h^2}{2\pi m_e kT_e}\right)^{3/2}\frac{g_n}{2}\exp\left(\frac{h\nu}{n^2kT_e}\right), \tag{8}$$

where h is Planck's constant, m_e is the electron mass, and $N(\mathrm{H\ II})$ is the concentration of ionized hydrogen.

We will assume that all line broadening is due to the Doppler effect, so that

$$\phi_{nm}(\nu) = \frac{1}{\pi^{1/2}\Delta\nu_{\mathrm{D}}}\exp\left[-\frac{(\nu - \nu_L)^2}{(\Delta\nu_{\mathrm{D}})^2}\right], \tag{9}$$

where ν_L is the frequency at the center of the line and $\Delta\nu_{\mathrm{D}}$ is the Doppler width, which is related to the line width at half intensity, $\Delta\nu_L$, by

$$\Delta\nu_L = 2\sqrt{(\ln 2)}\Delta\nu_{\mathrm{D}} \tag{10}$$

and to the dispersion velocity, V, by

$$\Delta\nu_{\mathrm{D}} = \nu_L V/c\ , \tag{11}$$

where c is the speed of light.

As has been shown by Goldberg (1966), the non-LTE line coefficients can be expressed in terms of the b_n's, where $b_n = N_n/(N_n)_{\mathrm{LTE}}$, so that

$$\kappa_L = b_n\left[1 - \Delta n\frac{kT_e}{h\nu}\frac{d(\ln b_n)}{dn}\right]\kappa_L^{\mathrm{LTE}} \tag{12}$$

and

$$j_L = b_m j_L^{\mathrm{LTE}}\ , \tag{13}$$

where $\Delta n = m - n$. As was first pointed out by Goldberg (1966), the quantity $d(\ln b_n)/dn$ is of great importance in determining κ_L, since it is usually large enough so that κ_L is negative. In the calculations made in this paper, we will use the data on $b_n = b_n(T_e,N_e)$ as calculated by Sejnowski and Hjellming (1969) for a range of values of T_e and N_e.[1]

Because the absorption and emission coefficients are dependent upon T_e and N_e, the solution for T_L and T_C by equations (1) and (2) is dependent upon knowing T_e and N_e at all points along the line of sight. This is obtained from the calculations discussed in § III.

[1] A recent paper by Dyson (1969) contains b_n calculations which are in essential agreement with those obtained by Sejnowski and Hjellming (1969); the differences in the two sets of results are shown by Sejnowski and Hjellming (1969) to be caused by an error introduced by Dyson (1969) in an approximation in the so-called cascade term in the equations of statistical equilibrium.

III. COMPUTATION OF VARIATIONS OF T_e AND N_e

If it is assumed that the gas in an H II region is locally in a state of ionization and thermal equilibrium, then the state of ionization and the electron temperature depend only on the local values of the density and the Lyman-continuum radiation field (cf. Hjellming 1966, 1968*a*). Therefore, if one specifies the input radiation field of the Lyman continuum and chooses an arbitrary density distribution, one can calculate T_e and N_e at every point in the nebula.

For the purposes of the present paper, we are interested only in the values of T_e and N_e along a line of sight through the center of a nebula. We will assume a particular density distribution which is symmetrical with respect to the center; in all, four different distributions are considered. In what we will call the A series of models, one has

$$N_{\rm H}(R) = N_0(1 - 0.9R/R_s) , \tag{14}$$

where $N_{\rm H}(R)$ is the hydrogen concentration at a distance R from the center, N_0 is an arbitrarily chosen constant, and R_s is the radius of the H II region. The constant N_0 is chosen to give particular values of the emission measure for the line of sight through the center of the nebula; if the emission measure is 10^5 pc cm^{-6}, then the model is labeled A5; if it is 10^6, then the model is labeled A6, etc.

TABLE 1

PROPERTIES OF EXCITING STARS

M_*/M (1)	$T_{\rm eff}$ (°K) (2)	R_*/R (3)	L_* (sec^{-1}) (4)	Q (5)
60	51050	9.705	3.57×10^{49}	0.406
45	47980	8.299	1.83×10^{49}	0.363
30	43550	6.745	6.87×10^{48}	0.270

In order to introduce the effects of clumping in varying degrees, a simple model was assumed in which twenty clumps are placed at equally spaced intervals along the line of sight and each clump was assumed to have the shape of a Gaussian. The resulting density distribution was superimposed upon a smooth density distribution so that

$$N_{\rm H}(R) = N_0(1 - 0.9R/R_s)\prod_{n=1}^{20}\{1 + 99 \exp[-(x - x_n)^2/(\Delta x)^2]\Big/(\pi^{1/2}\Delta x)\} , \tag{15}$$

where $x = R/R_s$, Δx is a free parameter, $x_n = 0.1\, n\, R/R_s$ where $n = 0, 1, \ldots, 10$, and the number 99 is a clumping amplitude representing cases where the density in the peak of a clump is 100 times the unclumped density. Three series of models with clumping are considered, with values of $\Delta x = 0.012$, 0.006, and 0.002 for the B, C, and D series of models, respectively.

In order to obtain theoretical models which will roughly correspond to the level of excitation of H II regions like W49, the stellar Lyman-continuum radiation will be taken to be that radiation provided by six massive stars assumed to be on the zero-age main sequence: one 60 $M_\odot$ star, two 45 $M_\odot$ stars, and three 30 $M_\odot$ stars. The properties of the Lyman continuum for these stars are taken from Hjellming (1968*b*) and are listed in Table 1. Column (1) gives the mass of the star, column (2) gives the effective temperature, column (3) gives the radius, column (4) gives the total photon luminosity in the Lyman continuum, L_*, and column (5) gives the fraction of L_* which can ionize helium. With this set of exciting stars, the total stellar input of the Lyman continuum is 9.3×10^{49} photons sec^{-1}, giving an excitation parameter for the model nebula of 134 pc cm^{-2}, if the temperature used in the formula for the excitation parameter (Hjellming 1968*b*) is taken to be 8000° K.

The diffuse radiation field of the Lyman continuum produced by recombinations of hydrogen and helium to the ground states should also be considered. Rubin (1968) carried out accurate computations showing that this radiation may be neglected near the center of an H II region, but in the outer regions the mean free paths of diffuse photons are short enough so that one can use the approximation that the intensity is j_ν/κ_ν, where j_ν and κ_ν are the mass emission and absorption coefficients for the diffuse radiation. Based upon the detailed calculations of Rubin (1968), the following interpolation formula has been found to provide satisfactory representation for the effects of the diffuse field:

$$I_\nu(\text{diffuse}) = \frac{j_\nu}{\kappa_\nu}\{1 - \exp[-(\tau_1/0.57)^{0.7}]\} , \tag{16}$$

where τ_1 is the optical depth at the Lyman limit.

With these specifications for the radiation fields and density distributions, the models were computed with the methods discussed by Hjellming (1966), with collision-strength

TABLE 2

PROPERTIES OF MODEL H II REGIONS

Model (1)	E (2)	$\langle N_e\rangle_{rms}$ (cm^{-3}) (3)	$\langle N_e\rangle_E/\langle N_e\rangle_{rms}$ (4)	$\langle T_e\rangle_{neb}$ (°K) (5)	$\langle T_e\rangle_{mass}$ (°K) (6)	$\langle T_e\rangle_E$ (°K) (7)
A5.......	9.92×10^4	68.0	1.24	8350	7020	6650
A6.......	9.89×10^5	378.4	1.24	8630	7410	7090
A7.......	9.90×10^6	2105.0	1.24	9030	7900	7600
D5......	1.15×10^5	74.9	6.36	8340	7250	7030
D6......	1.17×10^6	422.9	6.36	8790	7760	7570
D7......	1.19×10^7	2382.0	6.36	9190	8130	7900

parameters for the cooling ions taken from Seaton (1968), and with the following abundances: $N(\text{He})/N(\text{H}) = 0.1$, $N(\text{O})/N(\text{H}) = 0.001$, $N(\text{Ne})/N(\text{H}) = 0.0004$, and $N(\text{N})/N(\text{H}) = 0.00032$. The properties of these models for series A and D with emission measures of about 10^5, 10^6, and 10^7 are presented in Table 2. In this table column (1) contains the model designation and column (2) gives the emission measure, E (measured in pc cm^{-6}). One can define

$$\langle N_e\rangle_E = \int_0^s N_e^3 ds \Big/ \int_0^s N_e^2 ds \tag{17}$$

and

$$\langle N_e\rangle_{rms} = \left(\int_0^s N_e^2 ds \Big/ \int_0^s ds\right)^{1/2} . \tag{18}$$

The value of $\langle N_e\rangle_{rms}$ is given in column (3) of Table 2, and the ratio $\langle N_e\rangle_E/\langle N_e\rangle_{rms}$ is given in column (4). Averages of the temperature as defined by

$$\langle T_e\rangle_{neb} = \int_0^{R_s} T_e N_e R^2 dR \Big/ \int_0^{R_s} N_e R^2 dR , \tag{19}$$

$$\langle T_e \rangle_{\text{mass}} = \int_0^s T_e N_e ds \Big/ \int_0^s N_e ds \,, \tag{20}$$

and

$$\langle T_e \rangle_{\text{E}} = \int_0^s T_e N_e^2 ds \Big/ \int_0^s N_e^2 ds \tag{21}$$

are presented in columns (5), (6), and (7), respectively. It is clear that any measurement, in which the properties of the radiation field alone allow determination of T_e or N_e, will give a result which is close to $\langle T_e \rangle_E$ or $\langle N_e \rangle_E$, as will be borne out by the results discussed in § V.

IV. LINE AND CONTINUUM INTENSITIES

The variations of T_e and N_e over the lines of sight through the centers of the model H II regions listed in Table 2 were used in conjunction with the equations of § II to calculate $(T_L/T_C)_{\text{LTE}}$ and T_L/T_C for the centers of the following lines: H94α, H109α, H126α, H158α, H137β, and H225γ. In all calculations it was assumed that $V/c = 6 \times 10^{-5}$. The results of the calculations are presented in Table 3. Column (1) lists the model designation, column (2) identifies the line transition, and columns (3) and (4) show the values of $(T_L/T_C)_{\text{LTE}}$ and T_L/T_C, respectively. All of the quantities refer to lines of sight through the centers of the model nebulae.

The differences between $(T_L/T_C)_{\text{LTE}}$ and T_L/T_C are seen to be fairly small for the ($E = 10^5$)-cases, but for the cases with larger emission measures, the non-LTE line enhancement becomes very important. If one considered only models with smooth density distribution (series A), then the enhancement for models with $E \gtrsim 10^6$ is larger by a factor of about 2 or more than any that is ever observed for an H II region with a similarly large emission measure (Dieter 1967; Gardner and McGee 1967; Mezger and Hoglund 1967; Williams 1967; Zuckerman *et al.* 1967; Gordon and Meeks 1967, 1968; Mezger and Ellis 1968).

The solution to this dilemma, which has been a strong argument against the applicability of non-LTE effects, is found in the introduction of clumping, as has been done for the B, C, and D series of models. For example, a comparison of clumping effects on the enhancement of the H109α line is presented in Table 4 for the A, B, C, and D series of models. In Table 4, column (1) identifies the model series, column (2) gives $\langle N_e \rangle_E / \langle N_e \rangle_{\text{rms}}$, and columns (3), (4), and (5) give T_L/T_C for values of $E = 10^5$, 10^6, and 10^7, respectively. It is obvious from Table 4 that increased clumping decreases the enhancement. Physically this occurs because $d(\ln b_n)/dn$ usually decreases with increasing density (see Sejnowski and Hjellming 1969), thus decreasing the enhancement due to non-LTE.

The idea that there is severe clumping in H II regions is not new, since it has been extensively discussed in connection with the Orion Nebula. Menon (1961, 1962, 1966) has discussed the comparison of radio data (which determines $\langle N_e \rangle_{\text{rms}}$) and optical data (which essentially measure $\langle N_e \rangle_E$) of Osterbrock and Flather (1959). From Menon's discussion, the ratio $\langle N_e \rangle_E / \langle N_e \rangle_{\text{rms}}$ is about 6.7 for the center of Orion. The fact that one must introduce clumping to this degree or greater in all theoretical models with large emission measure, in order to avoid predicting line enhancement considerably in excess of what is ever observed, indicates that any interpretation of line data for real H II regions should allow for the possible effects of clumping.

V. ANALYSIS OF LINE AND CONTINUUM DATA

Clearly, the interpretation of observational data on T_C and T_L/T_C would be very difficult if one must always use the general formulae discussed in § II. For this reason it is important to use theoretical models of H II regions to test the validity of analyzing

TABLE 3

Model Predictions and Inferred Temperatures

Model	Line	$(T_L/T_C)_{LTE}$	T_L/T_C	$(T_e)_{LTE}$	T_e'	T_e	$<T_e>_E$
A5	H94α	0.0836	0.0667	7930	6440	6550	6650
	H109α	0.0510	0.0403	6740	6480	6550	
	H126α	0.0315	0.0387	5490	6600	6550	
	H158α	0.0148	0.0287	3670	6780	6530	
	H137β	0.0140	0.0147	6250	6560	6550	
	H225γ	0.0019	0.0026	5030	6750	6520	
A6	H94α	0.0770	0.0932	5950	7030	7000	7090
	H109α	0.0468	0.0787	4480	7160	6980	
	H126α	0.0285	0.0685	3210	7270	6960	
	H158α	0.0126	0.0555	-	7380	6930	
	H137β	0.0128	0.0185	5150	7190	6930	
	H225γ	0.0017	0.0033	3460	7320	6940	
A7	H94α	0.0686	0.182	3090	7780	7470	7600
	H109α	0.0399	0.164	-	7880	7470	
	H126α	0.0217	0.143	-	7940	7410	
	H158α	0.0050	0.0943	-	8020	7290	
	H137β	0.0109	0.0271	2910	7850	7440	
	H225γ	0.0007	0.0040	-	8310	6650	
D5	H94α	0.0780	0.0712	7500	6160	6980	7030
	H109α	0.0476	0.0484	6850	6740	6960	
	H126α	0.0294	0.0332	6250	7600	6920	
	H158α	0.0138	0.0191	5230	9080	6800	
	H137β	0.0130	0.0134	6780	7170	6950	
	H225γ	0.0018	0.0020	6310	8300	6860	
D6	H94α	0.0709	0.0810	6700	8060	7470	7570
	H109α	0.0431	0.0570	5900	9170	7400	
	H126α	0.0263	0.0412	5040	10220	7330	
	H158α	0.0116	0.0256	3030	11680	7180	
	H137β	0.0118	0.0138	6550	8960	7420	
	H225γ	0.0015	0.0020	5750	10160	7320	
D7	H94α	0.0652	0.111	4840	10750	7640	7900
	H109α	0.0376	0.0822	3340	11860	7560	
	H126α	0.0201	0.0606	-	12850	7450	
	H158α	0.0041	0.0312	-	14600	7090	
	H137β	0.0103	0.0158	5120	11190	7610	
	H225γ	0.0006	0.0014	-	14860	6780	

data for an H II region as if it were homogeneous in T_e and N_e. The basic criteria for validity of a method of analysis will be that any temperatures determined from the predicted line strengths should agree roughly with the value of $\langle T_e \rangle_E$ for the model.

For homogeneous nebulae, equations (1) and (2) can be integrated to obtain

$$T_L + T_C = T_e \frac{(1 + j_L/j_C)}{(1 + \kappa_L/\kappa_C)} \left\{1 - \exp\left[-\left(1 + \frac{\kappa_L}{\kappa_C}\right)\tau_C\right]\right\} \quad (22)$$

and

$$T_C = T_e[1 - \exp(-\tau_C)] \, . \quad (23)$$

The simplified forms of equation (23) in the optically thin and thick limits are well known; the corresponding optically thin and thick limits for T_L are:

$$T_L = b_m T_e \tau_L{}^{\mathrm{LTE}} \quad (24)$$

and

$$T_L = T_e \frac{(j_L/j_C - \kappa_L/\kappa_C)}{(1 + \kappa_L/\kappa_C)} \tau_C \, . \quad (25)$$

It can be shown from equation (25) that a line formed in LTE has $T_L = 0$ in the optically thick limit. Thus, $(T_L/T_C)_{\mathrm{LTE}}$ will approach zero as T_e approaches zero for any nebula; because of this, $(T_L/T_C)_{\mathrm{LTE}}$ always has a maximum at some temperature, and if one

TABLE 4

VARIATION OF ENHANCEMENT OF H 109α LINE WITH CLUMPING

Series	$\langle N_e \rangle_E / \langle N_e \rangle_{rms}$	T_L/T_C		
		$E = 10^5$	$E = 10^6$	$E = 10^7$
(1)	(2)	(3)	(4)	(5)
A.	1.24	0.0493	0.0787	0.164
B.	2.59	0.0499	0.0678	0.124
C.	3.65	0.0495	0.0636	0.104
D.	6.36	0.0484	0.0570	0.082

observes T_L/T_C for a nebula and interprets it according to LTE theory, there are always two possible solutions for the temperature. In practice one solution will always give a low temperature ($T_e < 3000°$ K); therefore, the usual practice of considering only the higher-temperature solutions is equivalent to asserting that one "knows" that the low-temperature solutions are wrong. Fortunately, this problem does not exist when a non-LTE analysis is used.

The predicted T_L/T_C presented in column (4) of Table 3 have been used to derive temperatures from equations (22) and (23) in three ways; these temperatures are compared with the actual value of $\langle T_e \rangle_E$ for the model listed in column (8). First of all, the assumption of LTE line formation was made whereby $b_n = 1$; and equations (22) and (23) were used to derive $(T_e)_{\mathrm{LTE}}$, which is given in column (5). It may be noted that for the cases in which $E = 10^5$ the LTE temperatures differ for different lines, but on the average they give results close to the actual $\langle T_e \rangle_E$. It is seen that $(T_e)_{\mathrm{LTE}}$ tends to decrease slowly as n increases for the α-lines. For the cases where $E \geq 10^6$, one finds $(T_e)_{\mathrm{LTE}}$ to be significantly less than $\langle T_e \rangle_E$. In some cases either the lines are stronger than possible with an LTE theory or the double-valued nature of the solution is such that discrimination between high- and low-temperature solutions is unreasonable.

Second, a form of non-LTE analysis was used whereby equations (22) and (23) are used in conjunction with b_n's evaluated in the sense $b_n = b_n(T_e, \langle N_e \rangle_{\mathrm{rms}})$. This corresponds to a non-LTE analysis which neglects all clumping effects; the temperatures obtained in this way are denoted by T'_e and are listed in column (6) of Table 3. One notes that for the cases in which $E = 10^5$, the values of T'_e for the different lines are much closer to one another and closer to the actual values of $\langle T_e \rangle_E$ than was the case for $(T_e)_{\rm LTE}$; however, as E gets larger, the value of T'_e begins to exceed considerably $\langle T_e \rangle_E$. This effect is most extreme for the cases where clumping is introduced.

Finally, a non-LTE analysis solving for temperatures based upon equations (22) and (23) and setting $b_n = b_n(T_e, \langle N_e \rangle_E)$ was used to obtain the temperatures T_e listed in column (7) of Table 3. It is seen that in nearly all cases, irrespective of the value of E, the temperatures determined from different lines are very similar and are generally no more than a few hundred degrees less than the actual $\langle T_e \rangle_E$ for the models. The only exceptions occur when the optical depth becomes large and radiation is "received" from only a part of the nebula. Thus, one would expect the temperatures obtained to differ, since the "averaging" is over a different range of conditions.

It is important to note that, in carrying out the solutions for temperature for the model H II regions, we used the known values of E and $\langle N_e \rangle_{\rm rms}$ or $\langle N_e \rangle_E$, which allowed us to solve for the temperature as the only unknown. In analyzing data for real H II regions, one may be able to determine E independently (from continuum data), but a value of $\langle N_e \rangle_E$, which should always be greater than $\langle N_e \rangle_{\rm rms}$, could be determined only if one either assumed a value for $\langle N_e \rangle_E / \langle N_e \rangle_{\rm rms}$ or insisted that a correct solution for both $\langle N_e \rangle_E$ and T_e could be obtained by imposing the condition that all radio recombination lines should, within the error limits, infer the same temperature.

VI. CONCLUSIONS

On theoretical grounds it is clear that knowledge of the strengths of radio recombination lines emitted by the same column of matter in an H II region should allow one to obtain a unique solution for the average temperature, $\langle T_e \rangle_E$; however, a non-LTE analysis, approximating the nebula as homogeneous, should be used in which the parameter $\langle N_e \rangle_E$, not $\langle N_e \rangle_{\rm rms}$, is used in evaluating the b_n's. If this is true for theoretical models about which we have complete knowledge of the structure, then it should also be true for data on real H II regions for which we know only the line and continuum intensities. Cases where a unique solution cannot be obtained for all lines (within reasonable error limits) should be explainable in terms of the effects of large optical depths; if this is not possible, non-LTE effects[2] and inhomogeneities in the gas should not be blamed for the discrepancy.

Non-LTE effects should certainly be important in determining the radio emission-line properties of H II regions with $E \gtrsim 10^6$; in such H II regions, many of which are known not to have extremely large line enhancement, it seems likely that it will be necessary to consider values of $\langle N_e \rangle_E$ which are much larger than $\langle N_e \rangle_{\rm rms}$, which is equivalent to allowing for a large degree of clumping. Data on the α-, β-, and γ-lines in such H II regions (e.g., Orion A, M17) are, at the present time, not easily comparable in terms of the theory of this paper because of the isolated fashion in which most of the lines have been measured by different observers using different equipment and different techniques. Current efforts to carry out simultaneous observations of α-, β-, and γ-lines with multichannel receivers covering at least two of the lines for which one wishes to compare intensities will allow one to make a reasonable comparison with theoretical predictions.

Finally, it should be noted that the clumping which must be introduced in models with $E \gtrsim 10^6$, in order to avoid excess line enhancement, is so extreme that these models begin to appear more like accumulations of dense, compact clouds in a low-density gas

[2] We are assuming that there are no serious errors in the b_n's. If the b_n's depended upon more than two-body radiative and collisional processes, both bound-bound and bound-free, then better b_n calculations would be necessary. This might be true if a proton and an electron cannot be treated as an isolated system during the recombination process.

rather than relatively smooth, gaseous distributions. If this is true, the physics of such H II regions becomes enormously more complicated.

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