Sources of gravity waves

Although gravitational signals from supernovae and black holes are estimated to be below the noise level of present detectors, they may soon be observed with new generation gravity telescopes.

Terrence J. Sejnowski

The discovery of many possible sources of gravitational radiation, such as neutron stars, compact x-ray sources, rapidly rotating binary novae and the violent events occurring in quasars and galactic nuclei, has opened a new era of general-relativity physics. In the 1960's nearly all workers in the field considered the detection of gravitational radiation "exceedingly unlikely." From this pessimistic estimate the balance of opinion has shifted to the optimistic view that gravitational radiation, a crucial prediction of Einstein's general theory of relativity, is now within reach of current experimental techniques.

Pioneering work

Joseph Weber, the pioneer in the field of gravitational wave detection, has been reporting coincident events in his detectors since 1969. An international effort to duplicate Weber's experiment has so far failed to confirm his results, although Weber continues to measure coincidences. Nonetheless, gravitational-wave detectors are improving dramatically and may soon be sensitive enough to detect gravitational radiation from known astrophysical sources. These advances are all based on Weber's original research on the theory of gravitational-wave detection and his construction of detectors to measure the dynamical curvature of space with unprecedented sensitivity.

Weber has mentioned to me that he was originally motivated by the challenge of measuring the Riemann curvature tensor, an abstract mathematical object that, we will soon see, plays a central role in Einstein's geometrical theory of gravity. Weber pursued his goal in spite of widespread skepticism, although he acknowledges strong encouragement from such supporters as P. A. M. Dirac, Freeman J. Dyson, J. Robert Oppenheimer and John A. Wheeler.

Weber's experiment sets an important upper bound at 1660 Hz, the resonant frequency of his detector. Present detectors are capable of measuring relative displacements of less than $10^{-15}$ cm (two orders of magnitude less than nuclear diameters) between the ends of a bar meters long. Improvements during the next decade should allow measurement of strains as small as $\delta l/l = 10^{-20}$. This is a good opportunity to examine the prospects of gravitational-wave detection, putting into perspective the theoretical basis, possible astrophysical sources, and the state of the experimental art of gravitational-wave detectors, which Jonathan Logan discussed in his recent PHYSICS TODAY article.

Gravitational radiation theory

During the years 1906-16 Einstein created a new theory of gravitation based on only a few general physical principles. In the great tradition of 19th-century field theory, he was inspired by a dissatisfaction with Newtonian "action at a distance." Remarkably, the theory, without an arbitrary parameter, made predictions of departures from Newtonian gravity that were subsequently verified.

It is ironic, however, that the part of Einstein's theory that would seem most essential to its field nature, the prediction of gravitational radiation, was the farthest from direct experimental verification; furthermore, there ensued until quite recently debate among workers in the field as to the very existence of gravitational radiation within the structure of the theory.

The issue of the theoretical existence of gravitational waves concerns us only in its resolution. Relativists have come to realize that the proper measure of the existence of gravitational waves is the Riemann curvature, $\mathcal{R}_{\alpha\beta\gamma\delta}$, a four-indexed tensor, which describes the local tidal forces. Previously, such technical problems as nonlinearity, lack of local gravitational energy density, and the general coordinate transformation group admitted by general relativity obscured the meaning of gravitational waves and even led Sir Arthur Eddington to the conclusion that some gravitational waves "travel at the speed of thought." (This was in reference to longitudinal waves, which, as he correctly understood, were only fictitious coordinate effects. Eddington did believe in transverse waves.)
Powerful methods were developed during the late 1950's and early 1960's to analyze the propagating curvature. The detector that Weber designed and constructed during the same period to measure the dynamical part of the curvature field is based on the concept of geodesic deviation (see the box on page 42). The main idea—It is not the position of a point that is important in relativity, but the separation of two nearby points.

Although the general theory of relativity is qualitatively distinguished from Maxwell's theory of electromagnetism by its nonlinear field equations, the complications that the nonlinearity introduces are only important in the presence of a very strong gravitational field, such as that around a compact object like a neutron star or a black hole. These complications arise when we ask what mechanisms might produce large amounts of gravitational radiation. This question will be discussed later, but for our present purposes we need only consider the weak-field limit of general relativity. The weak-field linear theory is certainly an excellent approximation in the solar system and in addition will share many important features with the exact general theory.

Moreover, the quantized linear theory can be thought of as a spin-two theory on flat space, which allows comparison with electromagnetism, a spin-one theory. Not only is the gravitational interaction weak to begin with, but dipole radiation, the dominant emission mode of electromagnetic radiation, is excluded in the general theory of relativity by the conservation of momentum. In electrodynamics a single charge can be accelerated by a non-electromagnetic force. There is no way to accelerate a mass without causing a gravitationally active disturbance elsewhere: all matter energy fields couple to gravity. Consequently gravitational dipole radiation is excluded, and the lowest radiation multipole is the quadrupole mode. Conversely, the gravitational wave energy is absorbed by matter via quadrupole coupling as reflected in the equation of geodesic deviation. (In the box, the deviation vector \( n^\mu \) has quadrupole moment \( Q_{ab} = m_{ab}q_{ab} \).) Both these facts contribute to the great difficulty we have generating and detecting gravitational radiation compared to the ease with which we can study electromagnetic radiation.

The absence of gravitational dipole radiation does not depend on the absence of negative gravitational mass (contrary to some early assertions). The key quantity is the gravitational "charge" to inertial mass ratio (active mass/passive mass). If this ratio is unity, the conservation of momentum excludes gravitational dipole radiation. The Brans–Dicke scalar tensor theory is a case where the ratio deviates from unity, thus allowing both dipole and scalar radiation. However, an extra factor involving the difference of the ratio from one (which is quite small) reduces the magnitude of scalar and dipole radiation in the Brans–Dicke theory to that expected for quadrupole radiation in the general theory.

The weakness of gravitational radiation is clearly illustrated in a comparison of radiative multipole solutions in gravity and electromagnetism. For the gravitational quadrupole mode the total radiated power, \( \dot{\varepsilon} \), at a frequency \( \omega \) is

\[
\frac{d\varepsilon}{dt} = -\frac{G}{45c^5}Q_{ab}Q^{ab} = -\frac{G\dot{\varepsilon}}{45c^5}Q^2 \tag{1}
\]

where \( Q_{ab} \) is the reduced quadrupole moment tensor for the stress energy distribution, \( c \) is speed of light and \( G \) is the gravitational constant. The radiated power from an electric dipole is

\[
\frac{d\varepsilon}{dt} = -\frac{2}{3\epsilon_0} \dot{\varepsilon}dE^2 = -\frac{2}{3\epsilon_0} \dot{\varepsilon}d^2 \tag{2}
\]

Therefore the ratio of power from gravitational quadrupole (\( Q = m^2 \)) to electric dipole (\( d = or \)) radiation is

\[
\frac{\dot{\varepsilon}_g}{\dot{\varepsilon}_d} = \frac{1}{30} \left( \frac{G\omega^2}{c^3} \right) hr \tag{3}
\]

where \( k = \omega/c \). The first factor is the ratio of gravitational to electric coupling, which for two electrons is the ratio of the gravitational attraction to electrical repulsion (\( = 10^{-42} \)). The second factor is a further reduction due
Geodesic deviation and the principle of detection

The physics of gravitational-wave detection starts with the deviation of two nearby geodesics (part A of the figure in box), measured by a small vector \( \eta^a \) connecting them. The figure shows two nearby geodesics (greatly exaggerated) with separation measured by light signals (the colored 45-degree lines). The proper time between ticks along the geodesic is a measure of the separation vector \( \eta^a \).

The time dependence of \( \eta^a \) is governed by the equation of geodesic deviation:

\[
\frac{d^2 \eta^a}{d \tau^2} + R_{abc}^d \eta^b \eta^c \eta^d = 0
\]

where \( \tau \) is the proper time along one of the geodesics and \( \eta^a \) is the tangent 4-velocity.

Two nearby masses in free fall will follow this law, and a measurement of \( \eta^a \), as for example by measuring laser fringe shift with a Michelson interferometer, will determine certain components of the curvature \( R_{abcd} \).

If we attach a spring between the two masses, as in part B of the figure, the right-hand side of the equation above is modified by the addition of a force term. The equation then is precisely the equation of a simple harmonic oscillator being driven by an external force.

\[
\eta^a + h \eta^a = - R_{abc} \eta^b \eta^c \eta^d
\]

The detectors designed by Joseph Weber are based on a similar analysis applied to an extended elastic body. In his experiment the effective deviation is measured by piezoelectric crystals bonded to aluminum bars.

to the quadrupole nature of gravitational radiation and varies as (source size/wavelength).2

Another important difference between electromagnetic and gravitational radiation is their state of polarization. Both are massless fields and hence have only two polarization states, but the tensor character of gravity is manifest in its polarization. Figure 1 shows the force field lines for the two independent states of polarization for gravitational waves, which can be compared with the usual pair of orthogonal vector states of polarization for electromagnetic radiation.

The polarization of a gravitational wave is a unique signature whose careful measurement would test its spin-two character as well as provide important information about the source of radiation. In addition such a measurement might rule out some of the numerous theories that differ from relativity but agree with it to first order for all the classical solar-system tests.2

In the case of a binary star (with equal masses in circular orbits) the second factor in equation 3 can be reduced with Kepler's Law to

\[
(q \eta^a)^2 = \frac{m^*}{r}
\]

where \( m^* = 20m/c^2 \) is the gravitational radius (the radius of a black hole with the same mass). As an example, the binary i Boo has period \( \tau = 0.3 \) day, \( m^*/r = 1 \text{ km}/3 \times 10^6 \text{ km} \approx 3 \times 10^{-7} \), and should radiate \( L \approx 2 \times 10^{30} \text{ erg/sec} \) in gravitational radiation corresponding to an apparent flux on the earth of \( 10^{-21} \text{ erg/cm}^2 \text{ sec} \). If there were no other forces acting on i Boo it would spiral down in \( 2 \times 10^6 \) years.

The calculation of gravitational radiation from a binary star was first made by Einstein in 1916. (Though correct in order of magnitude he erred in using an incorrect quadrupole-moment tensor.) Einstein concluded that:

"[The loss of energy from gravitational radiation] is so small that in all conceivable cases, it has a negligible practical effect."

Such a position was entirely reasonable at the time, because such compact objects as white dwarfs and neutron stars had not yet been discovered, and the shortest binary period then known was measured in days. Since then binaries with periods measured in minutes have been discovered (the binary HZ 29 has a period of 1051 seconds); this makes an enormous difference in the amount of energy radiated by gravitational waves. The \( \omega^6 \) dependence of equation 1 increases the loss rate by a factor of \( 10^{12} \). These binaries were discovered during a study of dwarf novae.6 These stars flicker, have occasional nova outbursts, and exhibit with unusual light curves. The model developed to explain the observations was a binary composed of a low-mass main-sequence star and a white dwarf, and characterized by the flow of mass from the former to the latter. This model, shown in -figure 2, qualitatively explained all the features described above, but there was no known mechanism to explain the required flow of mass.
gravitational radiation, which diminishes with distance by the inverse square law. Table 1 is a summary of possible sources; the estimates should be considered upper bounds of only rough order of magnitude. Excellent references for a complete review of this area are the article "Gravitational Wave Astronomy" by William Press and Kip Thorne and the recent book Gravitation by Charles Misner, Kip Thorne and John Wheeler.

A dimensionless measure of the gravitational wave strength is the departure of geometry from a flat Minkowski background:

$$g_{ab} = \eta_{ab} + h_{ab}$$

where the metric $g_{ab}$ measures proper space–time displacements: $ds^2 = g_{ab}dx^a dx^b$. In terms of $h_{ab}$ (or more correctly, its transverse traceless part) the curvature, ignoring indices, is

$$R = \frac{1}{c^2} h$$

and the energy flux density is

$$F = L_0 \left( \frac{h}{c} \right)^2$$

where $L_0 = c^3/G = 3.6 \times 10^{38}$ erg/sec is the natural unit of gravitational power. The magnitude of $h$ is a measure of the strain $\epsilon = \delta l/l$.

Astronomical bodies with high-velocity motion and strong gravitational fields are the most promising places to look for the generation of gravitational radiation. Good candidates are white dwarfs, neutron stars and black holes. These very dense objects have been discovered in short-period binaries, pulsars and compact x-ray sources. Riccardo Giacconi, at the I.A.U. Symposium in Warsaw in September 1973 reviewed the evidence that the x-ray source Cygnus X-1 is a black hole. If, as appears, the mass of the compact object is greater than six solar masses, the object could not have avoided gravitational collapse. Furthermore, there are half a dozen other candidates for black holes, and it is likely that some of these double-star systems, unlike Cygnus X-1, will have the proper orientation relative to us for occultation measurements. More evidence is needed before we can be certain that black holes have been observed.

Gravitational collapse. There should be a magnificent burst of gravitational radiation from a large star that dies and undergoes gravitational collapse. Some of the star's mass will be blown off in a violent supernova explosion, and the rest will contract to either a white dwarf, a neutron star or a black hole, depending on the amount of mass left in the core. Unfortunately, the details of gravitational collapse are little known, owing in part to complications such as rotation, nonspherical motions and core fragmentation. There is a variety of possibilities, but
each type of collapse should give off gravitational waves with a unique pulse shape. Much work must be done on this problem before gravitational waves can be used to "diagnose" supernova events.

Binary stars. A strong burst of gravitational radiation is also expected from a binary star as the components spiral around in the last stage before collision. Weber and Reginald Clemens have studied the response of the University of Maryland detectors to this source and conclude they could measure such an event occurring within 10,000 light years of the sun.11

A rotating neutron star should be a source of gravitational waves when spinning fast enough to lose axial symmetry, as may well happen during its first few days of life. The flux of radiation emitted is crucially dependent on the neutron star's frozen-in quadrupole moment, which can only be given an upper bound. In addition to the gravitational radiation emitted continuously at the frequency of rotation of the neutron star, the infrequent "glitches" in the observed period, which are thought to be due to changes in the star's figure, might cause strong bursts of radiation. Estimates for the amount of radiation are very tentative because of uncertainties such as whether neutron stars have a solid core.

A black hole is characterized by its one-way property: Energy may enter but nothing can come out.4 An object unlucky enough to be captured by a black hole will produce a strong pulse of gravitational radiation before reaching infinite curvature and density at the origin. This unphysical singularity is an embarrassment in the classical theory of gravitation but could lead to interesting new effects in the quantized version, which is yet to be completed.

Matter accreting onto a black hole is expected to serve as a source of electromagnetic radiation through compressional heating. A black hole has two ways to provide energy: the gravitational pull on infalling matter, and its own kinetic energy of rotation. The problem of calculating in detail the burst of radiation expected for a plunge from orbit into a black hole is difficult, involving for the first time the analysis of gravitational waves travelling in very strong gravitational fields. The problem has been attacked by calculating linear perturbations on the non-flat Schwarzschild background metric.12 The calculations predict that, for a compact object of mass \( m \) falling radially into a black hole of mass \( M \), the fraction 0.01 \( m/M \) of the rest mass \( m \) will be converted into gravitational radiation in a time \( \tau \approx 10^{-4} \) \( M/M_\odot \) sec at a frequency \( \nu \approx 1/\tau \) and bandwidth \( \approx 1/\nu \). A similar analysis has not yet been completed for the rotating background, which may prove a somewhat more efficient source.

The possibility of gravitational synchrotron radiation from a particle in a circular orbit very close to a black hole has been proposed and studied.13 Unfortunately, the orbits are so highly energetic that no known natural astrophysical mechanism achieves the required injection energy. In addition, the beaming effect is not as strong for gravitational radiation as in the case of electromagnetic or scalar waves. Lastly, calculations predict that the body will be torn apart by tidal forces at the Roche limit. Although gravitational synchrotron radiation seems unlikely to be of practical importance astrophysically, it does illustrate the complexity of black-hole physics. Relativists are very excited by the possibility of such genuinely general-relativistic sources, because if they do exist, and if we can detect gravitational radiation from them, a new era of experimental general relativity lies ahead.

Gravitational wave astronomy

We can imagine that there will someday be a gravitational-wave astronomy6 to complement astronomy in the electromagnetic spectrum. The information yielded by a gravitational-wave detector could be as important as its construction is difficult. To begin we design our detectors as "blind" men, with only a slight idea of what the gravitational "sky" looks like. (In fact, since the earth is practically transparent to gravitational radiation, observatories with gravitational-wave telescopes will be able to scan all the heavens at all times.) Until we know which are the "first magnitude" gravitational-wave sources, we cannot optimally design detectors to study them.

The problem of trying to imagine what sources of gravitational radiation might exist before actually looking for them is roughly comparable to the plight of cloud-bound Venussians who must deduce, from the physical laws found on the surface of their planet, the structure of their universe. (This point of view is adopted by Richard Feynman in his Lectures on Gravitation, mimeographed notes, 1962-3. He shows how one can arrive at Einstein's geometric theory of relativity starting from the linear spin-two graviton field theory.) Choosing those that are most probable from all the possibilities imagined is a central problem in astrophysics and a difficult one even with the aid of astronomical evidence. Though we can make intelligent guesses it would be foolish to think we already know the most important as-

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**Table 1: Astrophysical Sources of Gravitational Radiation**

<table>
<thead>
<tr>
<th>Source</th>
<th>Spectrum</th>
<th>Flux (erg/cm² sec)</th>
<th>Strength (deviation of metric from flat space)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binary stars in our galaxy. Highly monochromatic waves. Best case known.</td>
<td>Period = 1 hour</td>
<td>( F \approx 10^{-10} )</td>
<td>( h \approx 10^{-20} )</td>
</tr>
<tr>
<td>Total flux from all binaries</td>
<td>Peak at ( P \approx 6 ) hours</td>
<td>( F \approx 10^{-7} )</td>
<td>( h &lt; 10^{-10} )</td>
</tr>
<tr>
<td>Pulsars: Crab (NP0532) Continuous &quot;glitch&quot;</td>
<td>( \nu = 60 ) Hz</td>
<td>( F &lt; 3 \times 10^{-13} )</td>
<td>( h &lt; 10^{-17} )</td>
</tr>
<tr>
<td>( \nu = 10^3 ) Hz</td>
<td>( F &lt; 1 )</td>
<td>( h &lt; 10^{-14} )</td>
<td></td>
</tr>
<tr>
<td>Supernovae and stellar collapse:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In our galaxy approximately once every 100 years, (perhaps more often), bursts of duration approximately ( 10^{-3} ) to 1 sec. From all galaxies out to the Virgo cluster of galaxies; once a month.</td>
<td>( \nu = 10^6 ) to ( 10^8 ) Hz</td>
<td>( F &lt; 10^{10} )</td>
<td>( h &lt; 10^{-17} )</td>
</tr>
<tr>
<td>Neutron star rapidly rotating during the few days of its life in our galaxy. Explosions in distant quasars and galactic nuclei.</td>
<td>( \nu = 10^7 ) Hz</td>
<td>( F &lt; 10^4 )</td>
<td>( h &lt; 10^{-10} )</td>
</tr>
<tr>
<td>Black hole in the center of our galaxy with mass ( \approx 10^8 ) to ( 10^9 ) M(_\odot) which &quot;swallows&quot; a star of solar mass. Short pulse.</td>
<td>Period = 100 days</td>
<td>( F \approx 10^{-11} )</td>
<td>( h \approx 10^{-21} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( F \approx 10^{-2} )</td>
<td>( h \approx 10^{-10} )</td>
</tr>
</tbody>
</table>

Current detectors can pick up signals with strength \( h \approx 10^{-12} \); to be improved to \( 10^{-18} \) next year and to \( 10^{-20} \) within ten years.

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Gravitational wave detector based on laser interferometry, constructed by Forward and Moss. The fringe shift between the two arms is measured in the central chamber; the accuracy corresponds to a displacement between the mirrors of approximately $10^{-15}$ cm. This type of detector has the advantage, over the resonant type, of broad-band sensitivity and good potential for future improvement.

Figure 3

Gravitational wave detectors can be used to detect scalar radiation (an experiment carried out by Weber) or purely spin-two waves. For example, a detector instrumented to detect scalar radiation will be insensitive to purely spin-two waves.

The ability of a detector to pull a signal out of the noise depends on the averaging time, $\tau$, and goes as $\tau^{-1/2}$. This motivates one of several figures of merit for detectors, the "displacement sensitivity," $S$,

$$S = \delta l(\tau)^{1/2} \text{ (units: cm/Hz}^{1/2}\text{)}$$

where $\delta l(\tau)$ is the minimum detectable displacement in time $\tau$.

The laser interferometer of Robert Forward and Gaylord Moss at the Hughes Research Laboratory in Malibu, California is an example of a weakly coupled system. Their detector, pictured in figure 3, measures the fringe phase displacement between two interferometer mirrors with an accuracy of $\delta l \approx 10^{-7}$ cm. They have made broadband measurements (from 1.3 to 20 kHz) at a strain sensitivity of $S/l = 2 \times 10^{-16}/\text{Hz}^{1/2}$. This figure can be improved by increasing the length of the interferometer arm, $l$, or increasing the power of the laser; their present system is now at the photon fluctuation noise limit $\delta l \approx (\lambda/N)^{1/2}$ where $\lambda$ is the laser wavelength and $N$ the number of photons in a measurement. A high-power laser with a very long multiple-pass baseline might detect known sources such as binary stars and distant supernovae (see Table 1). Baselines the order of the earth's diameter in length could be achieved in space.

Most of the gravitational-wave detectors being built today are based on Weber's original design. These consist of aluminum bars highly isolated from the environment and instrumented with piezoelectric transducers to measure the amplitude of the quadrupole mode of vibration. Resonant detectors with large $Q$ (the resonance quality factor, which is the number of cycles required for a ringing mode to damp to 1/e of its original energy) improve the sensitivity to long monochromatic wave trains, $(\Delta l \sim h(Q)/\tau)$ but at the expense of losing all information about the wave except a single Fourier component. The present resonant detector technology is limited by the thermal noise of the detector. The magnitude of this noise for a detector...
of mass \( m \), temperature \( T \) and resonant frequency \( \omega \) is

\[ H \approx \left( \frac{kT}{m\omega^2} \right)^{\frac{1}{2}} \tag{4} \]

If an external excitation is of duration shorter than the time scale of thermal fluctuation, (which is approximately the damping time), then displacements even smaller than the amplitude of thermal noise can be measured. Hence detectors with large \( Q \) are particularly sensitive to "hammer-blow" signals.

The next generation of gravitational-wave detectors, cooled Weber-type cylinders, are now being built. The reduced thermal noise should improve strain sensitivity to \( \sigma = 10^{-20} \), a figure comparable to the strains that known sources are expected to produce (Table 1). Coincidences between a network of such detectors might achieve sufficient sensitivity to map the gravitational-wave sky.

William Fairbank at Stanford University, William Hamilton at Louisiana State University, and Eduardo Amaldi, Guido Pizzella, and Giorgio Careri at the University of Rome, in three separate but nearly identical experiments, are well along in the construction of 10,000-pound cylinders. The bars will be cooled to liquid-helium temperature and ultimately to \( 3 \times 10^{-3} \) K by adiabatic demagnetization. They will be floated on a superconducting grid to minimize heat loss and spurious vibrations. A superconducting magnetic shield will prevent the pickup of electromagnetic noise. Superconducting accelerometers will measure the motion of the bar with minimum noise and maximum stability.

These first attempts to detect gravitational radiation can be compared to Karl Jansky's pioneer observations in radio astronomy, except that he used a dish antenna, whereas present workers are using the equivalent of a dipole stub. The first task at hand is to find the strongest signals; elaborations, such as interferometry, that would greatly increase the directional sensitivity, should follow as they have in other areas of astronomy. The pace at which this proceeds depends as much on persistence as it does on what sources exist—and, of course, on luck. Thorne expressed to me the prospects he felt may lie ahead:

"I look forward with particular excitement to the time, perhaps eight years hence, when detectors will study bursts of waves from supernovae in the Virgo cluster of galaxies (about three supernovae per year!). Such bursts will be a direct probe of the ends of stellar collapse, as well as a powerful tool for distinguishing between competing theories of gravitation."

At detectors become more sensitive they could, in time, map out an entirely new astronomical window onto our universe. In the meantime we can patiently anticipate the first detection of gravitational waves from a known astronomical source, an exciting event once considered "exceedingly unlikely."

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I am indebted to John A. Wheeler for generous help and support. Special assistance was provided by Anne Buttemheim, John Faulkner, Robert Forward, James Hartle, Jonathan Logan, Kip Thorne, Tony Tyson and Joseph Weber.

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